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# Rodrigues's Formulae for The Generalized Hypergeometric Polynomials of Three Variables

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***Abstract***: *In the present paper, the generalized hypergeometric polynomial Set $R_n(x_1,x_2,x_3)$ has been expressed in terms of $n^{th}$ derivatives of certain Lauricella functions of the superior orders, which called a Rodrigues formula. Many interesting new results may be obtained as particular cases on specializing the parameters. Out of these particular results some of them stand for well known polynomials and of them are believed to be new. These formulae are at most important for mathematicians, scientists and engineers.*

***AMS Subject Classification****—Special function-33Cxx*
***Keywords:*** *Rodrigues formula, Generalized hypergeometric function, Lauricella function, Generatiy relation.*

## 1. Introduction

Singh and Singh [1] defined the generalized hypergeometric polynomial set $R_n(x_1, x_2, x_3)$ by means of generating relation,

$$(1-\nu t)^{-\lambda}\left(1\text{-}\mu_1 x_2^{r_1} t^{r_1}\right)^{\text{-}\lambda_1}$$

$$F\left[\begin{matrix}(A_p);(C_u);(E_h)(G_m) \\ \qquad\qquad \mu x_1^{r_1} t, \mu_2 x_2^{-r_2} t^{r_2}, \mu_3 x_3^{-r_3} t^{r_3} \\ (B_q);(D_v);(F_k);(H_w)\end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} R_{n,r;r_1;r_2;r_3;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\mu;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}\left(x_1, x_2, x_3\right) t^n \qquad \dots (1.1)$$

where $\nu$, $\lambda$, $\lambda_1$, $\lambda_2$, $\lambda_3$ are real and r, $r_1$ are non-negative integer and $r_2$, $r_3$ are natural numbers.
The left hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchanall and Chaundy [2].

The polynomial set contains number of parameters, for simplicity we shall denote

$$R_{n,r;r_1;r_2;r_3;(B_q);(D_v);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\mu;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}\left(x_1, x_2, x_3\right)$$

by $R_n(x_1, x_2, x_3)$.

where $n$ denotes the order of the polynomial set.

After little simplification (1.1) gives

$$R_n\left(x_1, x_2, x_3\right) = \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{r_2=0}^{\left[\frac{n-r-r_1 s_1}{e_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1 s_1-r_2 s_2}{e_3}\right]}$$

$$\times \frac{\left[(A_p)\right]_{n-r-r_1 s_1-(r_2-1)s_1-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1 s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times \frac{\left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s(\lambda_1)_{s_1}\,\nu^s\mu_1^{s_1}\mu_2^{s_2}\mu_3^{s_3}}{\left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}\,s!\,s_1!\,s_2!\,s_3!}$$

$$\times \frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3}\,x_2^{r_1s_1+r_2s_2}}{(n-r-r_1s_1-r_2s_2-r_3s_3)!\,x_3^{r_3s_3}} \qquad \ldots (1.2)$$

**2. Notations**

I. (i) $(n) = 1, 2, 3, \ldots\ldots\ldots n-1, n.$

(ii) $(a_p) = a_1, a_2, a_3, \ldots\ldots\ldots a_p.$

(iii) $(a_p\,;\,i) = a_1, a_2, \ldots\ldots a_{i-1}, a_{i+1} \ldots\ldots\ldots a_p.$

II. (i) $[(a_p)] = a_1 \cdot a_2 \cdot a_3, \ldots\ldots\ldots a_p.$

(ii) $\left[(a_p)\right]_n = \prod_{i=1}^{p}(a_i)_n = (a_1)_n(a_2)_n \ldots\ldots (a_p)_n.$

III. (i) $\Delta(a,b) = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots + \frac{b+a-1}{a}.$

(ii) $\Delta_k(a;b) = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \ldots\ldots \left(\frac{b+a-1}{a}\right)_k$

$$= \prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k$$

(iii) $\Delta\left[m;(a_p)\right] = \prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k.$

IV. (i) $\Gamma\left[(a_p)\right] = \prod_{i=1}^{p}\Gamma(a_i).$

(ii) $\Gamma\left[(a_p);s\right] = \prod_{i=s+1}^{p}\Gamma(a_i).$

(iii) $\Gamma\left[a+\frac{(m)}{m}\right] = \prod_{r=1}^{m}\Gamma\left(a+\frac{r}{m}\right).$

(iv) $\Gamma\left[\Delta(a;b)\right] = \prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right).$

V. (i) $\Gamma_*(a \pm b) = \Gamma(a+b)\Gamma(a-b).$

(ii) $\Gamma_*(a+b) = \Gamma(a+b)\Gamma(a-b).$

VI. (i) $M_1 = \frac{\left[(A_p)\right]_n\left[(C_u)\right]_n\left(\mu x_1^{r_4}\right)^n}{\left[(B_q)\right]_n\left[(D_v)\right]_n\,n!}$

**3.Rodrigues's Formulae for $R_n$ ($x_1$, $x_2$, $x_3$)**

**A. For $r_2 > 1$ and $r_3 > 1$**

Since $\delta^{(r_2-1)r_4 n}\left[x_1^{nr_3r_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3}\right]$

$$=\frac{(nr_3r_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3)!\,x_1^{nr_4-rr_4-r_1r_4s_1-r_2r_3s_2-r_3r_4s_3}}{(nr_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3)!}$$

where $\delta\equiv\frac{\delta}{\delta x_1}$

Hence from (1.2), we have

$$R_n(x_1,x_2,x_3)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{s_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times\frac{[(A_p)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\,[(C_u)]_{n-r-r_1s_1-r_2s_2-r_3s_3}}{[(B_q)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\,[(D_v)]_{n-r-r_1s_1-r_2s_2-r_3s_3}}$$

$$\times\frac{[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}\,\mu_1^{s_1}\,\mu_2^{s_2}\,x_2^{r_1s_1+r_2s_2}\,\mu_3^{s_3}}{[(F_k)]_{s_2}\,[(H_w)]_{s_3}\,s!\,s_1!\,s_2!\,s_3!\,x_3^{r_3s_3}}$$

$$\times\frac{\mu^{n-r-r_1s_1-r_2s_2-r_3s_3}\,\delta^{(r_3-1)nr_4}\,x_1^{nr_3r_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3}}{(n-r-r_1s_1-r_2s_2-r_3s_3)!}$$

$$\times\frac{(nr_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3)!}{(nr_3r_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3)!}$$

Now, since

$$\delta^{(r_3-1)r_4n}\,x_1^{(nr_3r_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_3r_4s_3)}=0$$

For , $\frac{n}{r},\frac{n}{r_1},\frac{n}{r_2}$ and $\frac{n}{r_3}<k\le n$

Hence

$$R_n(x_1,x_2,x_3)=\frac{[(A_p)]_n\,[(C_u)]_n\,(nr_4)!\,\mu^n}{[(B_q)]_n\,[(D_v)]_n\,n!\,(nr_3r_4)!}\,\delta^{(r_3-1)r_4n}\left(x_1^{nr_4}\right)$$

$$\times\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}\frac{[1-(B_q)-n]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}}{[1-(A_p)-n]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}}$$

$$\times \frac{\left[1-(D_v)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}}{\left[1-(C_u)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}\; s! \quad s_1!}$$

$$\times \frac{\mu_1^{s_1}\;\mu_2^{s_2}\;\mu_3^{s_3}\;x_2^{r_1s_1+r_2s_2}\,(-n)_{r+r_1s_1+r_2s_2+r_3s_3}}{s_2!\;\;s_3!\;\;x_3^{r_3s_3g_1}\,(-n)_{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{(-nr_4r_3)_{r+r_1r_4s_1+r_2s_2r_4+r_3s_3r_4}\,(-1)^{(p+q+u+v+1)r}}{\mu^{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{(-1)^{(p+q+u+v+1)(r_1s_1+r_2s_2+r_3s_3)}}{\left(x_1^{r_4}\right)^{r+r_1s_1+r_2s_2+r_3s_3}} \qquad \ldots (3.1)$$

The single terminating factor $(-n)_{r+r_1s_1+r_2s_2+r_3s_3}$ makes all summation in (3.1) runs up to $\infty$.

$$R_n(x_1,x_2,x_3)=M_1\frac{(nr_4)!}{(nr_3r_4)}\delta^{(r_3-1)r_4n}x_1^{nr_4}\sum_{s=0}^{\infty}\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{\infty}\sum_{s_3=0}^{\infty}$$

$$\times \frac{\left[1-(B_q)-n\right]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}\left[1-(D_v)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}}{\left[1-(A_p)-n\right]_{r+r_1s_1+(r_2-1)s_2+(r_3-1)s_3}\left[1-(C_u)-n\right]_{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(-n)_{r+r_1s_1+r_2s_2+r_3s_3}\,(-nr_4r_3)_{r+r_1s_1r_4+r_2s_2r_4+r_3s_3r_4}}{\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}(-nr_4)_{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times \frac{(\lambda)_s\,\nu^s\,(-1)^{(p+q+u+v+1)r}\,(-1)^{(p+q+u+v+1)r_1s_1}\;x_2^{r_1s_1}\,(\lambda_1)_{s_1}\,\mu_1^{s_1}}{s!\quad \mu^r\;\left(x_1^{r_4}\right)^r\;\mu^{r_1s_1}\quad s_1!\;\left(x_1^{r_4}\right)^{r_1s_1}}$$

$$\times \frac{(-1)^{\{(p+q+u+v+1)r_2+p+q\}s_2}\;x_2^{r_2s_2}\mu_2^{s_2}\,(-1)^{\{(p+q+u+v+1)r_3+p+q\}s_3}\;\mu_3^{s_3}}{s!\quad \mu^{r_2s_2}\;\left(x_1^{r_4}\right)^{r_2s_2}\quad s_3!\;\;\mu^{r_3s_3}\left(x_3x_1^{r_4}\right)^{s_3r_3}}$$

$$=M_1\frac{(nr_4)!}{(nr_4r_3)!}\delta^{(r_3-1)nr_4}x_1^{nr_4}F_{1+p+u:k:w}^{2+q+v:h:m:1:1}\left[\begin{matrix}[-n:r,r_1,r_2,r_3]\\ [-nr_4:r,r_1,r_2,r_3]\end{matrix}\right.$$

$$\begin{matrix}[-nr_3r_4:r,r_1,r_2,r_3];\left[\left(1-(B_q)-n\right):r,r_1,r_2-1,r_3-1\right],\\ \overline{\qquad\qquad\qquad\qquad};\left[\left(1-(A_p)-n\right):r,r_1,r_2-1,r_3-1\right],\end{matrix}$$

$$\begin{matrix}\left[\left(1-(D_v)-n\right):r,r_1,r_2,r_3\right],\left[(E_h):1\right],\left[(G_m):1\right],[\lambda:1],[\lambda_1:1],\\ \left[\left(1-(C_u)-n\right):r,r_1,r_2,r_3\right],\left[(F_k):1\right],\left[(H_w):1\right],—,—,\end{matrix}$$

$$\frac{(-1)^{r(p+q+u+v+1)}\,v}{\left(\mu x_1^{r_4}\right)^{r}}, \frac{(-1)^{r_1(p+q+u+v+1)}\,\mu x_2^{r_1}}{\left(\mu x_1^{r_4}\right)^{r_1}},$$

$$\left.\frac{(-1)^{r_2(p+q+u+v+1)+p+q}\,\mu_2 x_2^{r_2}}{\left(\mu x_1^{r_4}\right)^{r_2}}, \frac{(-1)^{r_3(p+q+u+v+1)+p+q}\,\mu_3}{\left(\mu x_1^{r_4} x_3\right)^{r_3}}\right] \quad \ldots (3.2)$$

**Particular Cases**

(i) If we put $p = 0 = q = u = v = m$; $w = 1 = r = r_4 = v = \lambda = \mu = \mu_3 = r_3$; $H_1 = 1$, $r_3 = 2$ and $\frac{x}{\sqrt{x^2-1}}$ for $x_1$ in (3.2), we get

$$P_n(x) = \frac{n!}{2n!}\frac{d^n}{dx^n}\left\{x^n F\left[\begin{matrix}(-2n;2); \\ \\ 1;\end{matrix}\ \frac{x^2-1}{x^2}\right]\right\}$$

where $P_n(x)$ are the Legendre Polynomials.

(ii) On making the substitution $p = 0 = q = u = v$; $r = 1 = r_4 = v = \lambda = x_2$; $\mu_3 = \mu = v$, $r_3 = m$, $x_1 = x$ and replacing $(G_m)$ by $(a_r)$ and $(H_w)$ by $(b_s)$ in (3.2), we arrive at

$$A_n(x) = \frac{n!}{(mn)!}\frac{d^n}{dx^n}\left\{\frac{(vx)^n}{n!} F\left[\begin{matrix}\Delta(-mn;-m);(a_r); \\ \\ (b_s)\end{matrix}\ \mu\left(\frac{-m}{vx}\right)^m\right]\right\}$$

where $A_n(x)$ are the generalized polynomials defined by Panda [3].

(iii) On putting $p = 0 = q = u = v = s$; $r = 1 = r_4 = \lambda = v = \mu = \mu_3 = x_1 = x_2$; $r_3 = p = \mu$; $\mu_3 = x$ in (3.2) and $G_m = \alpha_u$; $H_n = \beta_v$; we get

$$\beta_n^p(x) = \frac{n!}{(pn)!}\frac{d^n}{dx^n}\left\{(-p)^n F\left[\begin{matrix}\Delta(-np;p);(\alpha_u); \\ \\ (\beta_v)\end{matrix}\ x\right]\right\}$$

where $B_n^p(x)$ are the generalization of Hermite polynomials by Brafman [4].

(iv) On putting $q = 0 = v = m = s$; $p = 1 = u = w = \lambda = v = x = r_4$; $\mu_3 = -1$, $r_3 = 2 = \mu$; $A_1 = 1$, $C_1 = v$, $H_1 = v$ and $z$ for $x_1$ in (3.2), we achieve

$$R_{n,v}\left(\frac{1}{z}\right) = \frac{n!(v)_n}{(2n)!}\frac{d^n}{dx^n}\left\{(2z)^n F\left[\begin{matrix}(-n;2); \\ \\ v,-n,1-v-n;\end{matrix}\ -\frac{1}{z^2}\right]\right\}$$

where $R_{n,v}\left(\frac{1}{z}\right)$ are the Lommel Polynomials [5]

(v) On taking $p = 0 = q = s$; $u = 1 = v = m = w = \lambda = v = r = r_4 = x_2 = y$, $r_3 = m$, $\mu_3 = \mu$; $\mu = v$ and instead $(D_u) = (\beta_q)$; $x_1 = x$ $(C_u) = (\alpha_p)$, $(G_m) = (a_r)$ and $(H_w) = (b_s)$ in (3.2), we arrive at

$$\beta_n(x,y) = \frac{n!}{(mn)!}\frac{\left[(\alpha_p)\right]_n}{\left[(\beta_q)\right]_n}\frac{d^n}{dx^n}$$

$$\times\left\{(\nu x)^{n} F\left[\begin{matrix}\Delta(m;-n),\Delta(m;1-(B_q)-n); \\ \dfrac{\mu(-m)^{m(q-p+1)}}{(\nu xy)^{m}} \\ \Delta(m;1-(\alpha_p)-n),(b_s)\end{matrix}\right]\right\}$$

where $B_n(x, y)$ are the Polynomials defined by Khanna I.K.[6]

**B. For $r_2 = 1$ and $r_3 > 1$**

Since $\delta^{nr_4}\left[\left(x_1^{r_4}\right)^{2n-r-r_1s_1-r_2s_2-r_3s_3}\right]$

$$=\frac{(2nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3)\,x_1^{nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3}}{(nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3)!}$$

where, $\delta=\dfrac{\delta}{\delta x}$

Hence from (1.2), we get

$$R_n(x_1,x_2,x_3)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{s_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}\frac{[(A_p)]_{n-r-r_1s_1-(r_3-1)s_3}}{[(B_q)]_{n-r-r_1s_1-(r_3-1)s_3}}$$

$$\times\frac{[(C_u)]_{n-r-r_1s_1-s_2-r_3s_3}\,[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}\,\mu_1^{s_1}}{[(D_v)]_{n-r-r_1s_1-s_2-r_3s_3}\,[(F_k)]_{s_2}\,[(H_w)]_{s_3}\quad s!\quad s_1!}$$

$$\times\frac{\mu_2^{s_2}\quad x_2^{r_1s_1+s_2}\quad \mu_3^{s_3}\quad \mu^{n-r-r_1s_1-s_2-r_3s_3}}{s_2!\quad s_3!\quad x_3^{r_3s_3}\quad (n-r-r_1s_1-s_2-r_3s_3)!}$$

$$\times\frac{(nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3)!\;\delta^{nr_4}\,x_1^{2nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3}}{(2nr_4-rr_4-r_1r_4s_1-r_4s_2-r_3r_4s_3)!}$$

Now, since

$$\delta^{nr_4}\left(x_1^{r_4}\right)^{2n-r-r_1s_1-s_2-r_3s_3}=0$$

For $\dfrac{n}{r},\dfrac{n}{r_1},n$ and $\dfrac{n}{r_3}<k\le 2n,$

Hence

$$R_n(x_1,x_2,x_3)=\frac{[(A_p)]_n\,[(C_u)]_n\,\mu^n\,(nr_4)!\;\delta^{nr_4}\,x_1^{2nr_4}}{[(B_q)]_n\,[(D_v)]_n\quad n!\;(2nr_4)!}$$

$$\times\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{s_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}\frac{[1-(B_q)-n]_{r+r_1s_1+(r_3-1)s_3}}{[1-(A_p)-n]_{r+r_1s_1+(r_3-1)s_3}}$$

$$\times\frac{[1-(D_v)-n]_{r+r_1s_1+s_2+r_3s_3}\,[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}}{[1-(C_u)-n]_{r+r_1s_1+s_2+r_3s_3}\,[(F_k)]_{s_2}\,[(H_w)]_{s_3}\;s!\quad s_1!}$$

$$\times \frac{\mu_1^{s_1}\ \mu_2^{s_2}\ x_2^{r_1 s_1+s_2}\ \mu_3^{s_3}\ \mu^{-r-r_1 s_1-s_2-r_3 s_3}\ (-n)_{r+r_1 s_1+s_2+r_3 s_3}}{s_2!\ \ x_3^{r_3 s_3}\ s_3!\ (-nr_4)_{r+r_1 s_1+s_2+r_3 s_3}}$$

$$\times \frac{(-nr_4 r_3)_{r+r_1 s_1 r_4+r_4 s_2+r_3 r_4 s_3}\ (-1)^{(p+q+u+v+1)r+r_1 s_1+s_2+r_3 s_3}}{\left(x_1^{r_4}\right)^{r+r_1 s_1+s_2+r_3 s_3}} \quad \ldots (3.3)$$

The single terminating factor $(-n)_{r+r_1 s_1+s_2+r_3 s_3}$ makes all summation in (3.3) runs up to ∞.

$$R_n(x_1,x_2,x_3)=M_1\frac{(nr_4)!}{(2nr_4)!}\delta^{nr_4}x_1^{2nr_4}\sum_{s=0}^{\infty}\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{\infty}\sum_{s_3=0}^{\infty}$$

$$\times\frac{\left[1-(B_q)-n\right]_{r+r_1 s_1+(r_3-1)s_3}\left[1-(D_v)-n\right]_{r+r_1 s_1+s_2+r_3 s_3}}{\left[1-(A_p)-n\right]_{r+r_1 s_1+(r_3-1)s_3}\left[1-(C_u)-n\right]_{r+r_1 s_1+s_2+r_3 s_3}}$$

$$\times\frac{\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(-n)_{r+r_1 s_1+s_2+r_3 s_3}(-2nr_4)_{rr_4+r_1 r_4 s_1+s_2 r_4+r_3 s_3 r_4}}{\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}(-nr_4)_{rr_4+r_4 s_1+r_4 s_2+r_3 s_3 r_4}}$$

$$\times\frac{(\lambda)_s\,\nu^s\,(-1)^{(p+q+u+v+1)r}\,(-1)^{(p+q+u+v+1)r_1}\,x_2^{r_1 s_1}\,(\lambda_1)_{s_1}\,\mu_1^{s_1}}{s!\ \left(\mu x_1^{r_4}\right)^r\ \left(\mu x_1^{r_4}\right)^{r_1 s_1}\ s_1!}$$

$$\times\frac{(-1)^{\{(p+q+u+v+1)r_2+p+q\}s_2}(\mu_2 x_2)^{s_2}(-1)^{\{(p+q+u+v+1)r_3+p+q\}s_3}\,\mu_3^{s_3}}{s_2!\ \left(\mu x_1^{r_4}\right)^{s_2}\ s_3!\ \left(\mu x_1^{r_4}x_3\right)^{r_3 s_3}}$$

$$=M_1\frac{(nr_4)!}{(2nr_4)!}\delta^{nr_4}x_1^{2nr_4}F^{2+q+v:h:m:1:1}_{1+p+u:k:w}\left[\begin{matrix}[-n:r,r_1,1,r_3]\\ [-nr_4:r,r_1,1,r_3]\end{matrix}\right.$$

$$[2nr_4:r,r_1,1,r_3];\left[(1-(B_q)-n):r,r_1,r_3-1\right],$$

$$\overline{\qquad\qquad};\left[(1-(A_p)-n):r,r_1,r_3-1\right],$$

$$\left[(1-(D_v)-n):r,r_1,1,r_3\right],\left[(E_h):1\right],\left[(G_m):1\right],[\lambda:1],[\lambda_1:1],$$

$$\left[(1-(C_u)-n):r,r_1,1,r_3\right],\left[(F_k):1\right],\left[(H_w):1\right],—,—,$$

$$\frac{(-1)^{r(p+q+u+v+1)}\,\nu}{\left(\mu x_1^{r_4}\right)^r},\frac{(-1)^{r_1(p+q+u+v+1)}\,\mu x_2^{r_1}}{\left(\mu x_1^{r_4}\right)^{r_1}},$$

$$\left.\frac{(-1)^{(p+q+u+v+1)+p+q}\,\mu_2 x_2}{\left(\mu x_1^{r_4}\right)},\frac{(-1)^{(p+q+u+v+1)r_3+p+q}\,\mu_3}{\left(\mu x_1^{r_4}x_3\right)^{r_3}}\right] \quad \ldots (3.4)$$

**Particular Cases**

(i) On setting $p = 0 = q = u = v = s$; $\lambda = 1 = \nu = \lambda_1 = r = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$ and writing $x$ for $x_1$ in (3.4) we achieve

$$H_n(x) = \frac{n!}{(2n)!}\frac{d^n}{dx^n}\left\{(2x)^n F\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2}+\frac{1}{2};(2n;2); \\ \qquad\qquad -\frac{1}{x^2} \\ (-n;2); \end{matrix}\right]\right\}$$

where $H_n(x)$ are the Hermite Polynomials.

(ii) If we set $q = 0 = s$; $p = 1 = u = v = \lambda = \lambda_1 = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$, $A_1 = \alpha$, $C_1 = \beta$, $D_1 = \alpha + \beta$ and writing $x_1$ for $x$ in (3.4), we get

$$G_n(\alpha,\beta,x) = \frac{1}{(2n)!}\frac{(\alpha)_n}{(2n)!}\frac{(\beta)_n}{(\alpha+\beta)_n}\frac{d^n}{dx^n}\times\left\{(2x^n)F\left[\begin{matrix} \Delta(-n,2);1-\alpha,\beta-n; \\ \qquad\qquad -\frac{1}{x^2} \\ (2n;2),1-\alpha-n,1-\beta-n; \end{matrix}\right]\right\}$$

where $G_m(\alpha, \beta, \xi)$ are Bedient Polynomials.

(iii) On setting $p = 0 = q = u = v = s$; $r = r_4 = 1 = \lambda = \nu = \lambda_1$; $\mu_1 = h$; $\mu = m^{-m}$, $r_1 = m$ and $x$ for $x_1$ in (3.4), we achieve

$$g_n^m(x,h) = \frac{n!}{(2n)!}\frac{d^n}{dx^n}\left\{x^n F\left[\begin{matrix} \Delta(m;-n), \\ \qquad h\left(-\frac{m}{x}\right)^m \\ \Delta(2n;2) \end{matrix}\right]\right\}$$

where, $g_n^m(x;h)$ are the Gould-Hopper Polynomials [7].

(iv) For $p = 0 = q = u = v = s = r_4 = r$; $r_1 = p$, $\mu_1 = -1$, $\lambda = 1 = \lambda_1 = \nu = \mu$; and writing $x$ for $x_1$ in (3.4), we have

$$g_n^p(x) = \frac{n!\,p^n}{(2n)!}\frac{d^n}{dx^n}\left\{x^n F\left[\begin{matrix} \Delta(p;-n), \\ \qquad -\left(-\frac{1}{x}\right)^p \\ \Delta(2n;2); \end{matrix}\right]\right\}$$

where $g_n^p(x)$ are the Bragg Polynomials[8].

(v) On making the substitution $p = 0 = q = u = v = s$; $r_4 = r = 1 = \lambda = \lambda_1 = \nu$; $\mu_1 = \beta$; $\mu = \alpha$; $x_1 = m = r_1$ in (3.4), we get

$$H_{n,m}(\alpha,\beta) = \frac{n!}{(2n)!}\frac{d^n}{dx^n}\left\{x^n F\left[\begin{matrix} \Delta(m;-n), \\ \qquad \frac{\beta}{\alpha}(-m)^n \\ \Delta(2n;-2); \end{matrix}\right]\right\}$$

where $H_{n,m}(\alpha,\beta)$ are the generalized polynomials defined by Gupta and Jain[9].

(vi) If we set $p = 0 = q = u = v = s$; $r_4 = 1 = \lambda = \lambda_1 = \nu$; $\mu_1 = -1$; $\mu = \nu$ and replacing $x_1$ by $x$ in (3.4), we achieve

$$H_{n,mN}(x) = \frac{n!\,\nu^n}{(2n)!}\frac{d^n}{dx^n}\left\{x^n F\left[\begin{matrix} \Delta(m;-n),(2n;m); \\ \qquad -\frac{(-m)^n}{\nu^n} \\ \text{——}; \end{matrix}\right]\right\}$$

where $H_{n,m,\nu}(x)$ are the generalized polynomials defined by M. Lahiri[10].

**C. For $r_2 > 1$ and $r_3 = 1$**

Since $\delta^{nr_4}\left[\left(x_1^{r_4}\right)^{2n-r-r_1s_1-r_2s_2-s_3}\right]$

$$=\frac{\left(2nr_4-rr_4-r_1r_4s_1-r_2r_4s_2-r_4s_3\right)\left(x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-s_3}}{\left(nr_4-r_4r-r_1r_4s_1-r_2r_4s_2-r_4s_3\right)!}$$

where, $\delta\equiv\frac{\delta}{\delta x_1}$

Hence from (1.2), we get

$$R_n\left(x_1,x_2,x_3\right)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{s_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}\frac{\left[\left(A_p\right)\right]_{n-r-r_1s_1-(r_2-1)s_2}}{\left[\left(B_q\right)\right]_{n-r-r_1s_1-(r_2-1)s_2}}$$

$$\times\frac{\left[\left(C_u\right)\right]_{n-r-r_1s_1-r_2s_2-s_3}\left[\left(E_h\right)\right]_{s_2}\left[\left(G_m\right)\right]_{s_3}(\lambda)_s\,\nu^s\left(\lambda_1\right)_{s_1}\mu_1^{s_1}}{\left[\left(D_v\right)\right]_{n-r-r_1s_1-r_2s_2-s_3}\left[\left(F_k\right)\right]_{s_2}\left[\left(H_w\right)\right]_{s_3}\quad s!\quad s_1!\quad s_2!}$$

$$\times\frac{\mu_2^{s_2}\quad x_2^{r_1s_1+r_2s_2}\quad\mu_3^{s_3}\quad\mu^{n-r-r_1s_1-r_2s_2-s_3}}{s_3!\quad x_3^{s_3}\quad\left(n-r-r_1s_1-r_2s_2-s_3\right)!}$$

$$\times\frac{\left(nr_4-rr_4-r_1s_1r_4-r_2s_2r_4-r_4s_3\right)!\ \delta^{nr_4}x_1^{(2n-r-r_1s_1-r_2s_2-s_3)r_4}}{\left(2nr_4-r_4r_1-r_1r_4s_1-r_2r_4s_2-r_4s_3\right)!}\quad\ldots(3.5)$$

Now, since

$$\delta^{nr_4}\left(x_1^{nr_4}\right)^{2n-r_1-r_1s_1-r_2s_2-s_3}=0$$

For $\frac{n}{r},\frac{n}{r_1},\frac{n}{r_2}$ and $n<k\le 2n$

Hence,

$$R_n\left(x_1,x_2,x_3\right)=\frac{\left[\left(A_p\right)\right]_n\left[\left(C_u\right)\right]_n\mu^n\left(nr_4\right)!\ \delta^{nr_4}x_1^{2nr_4}}{\left[\left(B_q\right)\right]_n\left[\left(D_v\right)\right]_n\quad n!\ \left(2nr_4\right)!}$$

$$\times\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{s_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{1}\right]}\frac{\left[1-\left(B_q\right)-n\right]_{r+r_1s_1+(r_2-1)s_2}}{\left[1-\left(A_p\right)-n\right]_{r+r_1s_1+(r_2-1)s_2}}$$

$$\times\frac{\left[1-\left(D_v\right)-n\right]_{r+r_1s_1+r_2s_2+s_3}\left[\left(E_h\right)\right]_{s_2}\left[\left(G_m\right)\right]_{s_3}(\lambda)_s\,\nu^s\left(\lambda_1\right)_{s_1}}{\left[1-\left(C_u\right)-n\right]_{r+r_1s_1+r_2s_2+s_3}\left[\left(F_k\right)\right]_{s_2}\left[\left(H_w\right)\right]_{s_3}\ s!\quad s_1!}$$

$$\times\frac{\mu_1^{s_1}\ \mu_2^{s_2}\ x_2^{r_2s_2+r_1s_1}\ \mu_3^{s_3}\ (-n)_{r+r_1s_1+r_2s_2+s_3}}{s_2!\quad x_3^{r_3s_3}\ s_3!\ \mu^{r+r_1s_1+r_2s_2+r_3s_3}}$$

$$\times\frac{\left(-nr_4r_3\right)_{r+r_1s_1r_4+r_4s_2r_2+r_4s_3}\left(-1\right)^{(p+q+u+v+1)(r+r_1s_1+r_2s_2+s_3)}}{\left(-nr_4\right)_{r+r_1s_1+r_2s_2+s_3}\quad\left(x_1^{r_4}\right)^{r+r_1s_1+r_2s_2+s_3}}\quad\ldots(3.6)$$

The single terminating facter $(-n)_{r+r_1s_1+r_2s_2+s_3}$ makes all summation in (3.6) runs up to $\infty$.

$$R_n(x_1,x_2,x_3)=\frac{\mu_1(nr_4)!}{(2nr_4)}\delta^{nr_4}x_1^{2nr_4}\sum_{s=0}^{\infty}\sum_{s_1=0}^{\infty}\sum_{s_2=0}^{\infty}\sum_{s_3=0}^{\infty}$$

$$\times\frac{[1-(B_q)-n]_{r+r_1s_1+(r_2-1)s_2}\,[1-(D_v)-n]_{r+r_1s_1+r_2s_2+s_3}}{[1-(A_p)-n]_{r+r_1s_1+(r_2-1)s_2}\,[1-(C_u)-n]_{r+r_1s_1+r_2s_2+s_3}}$$

$$\times\frac{[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(-n)_{r+r_1s_1+r_2s_2+s_3}\,(-2nr_4)_{rr_4+r_1r_4s_1+r_2r_4s_2+s_3r_4}}{[(F_k)]_{s_2}\,[(H_w)]_{s_3}\,(-nr_4)_{rr_4+r_1r_4s_1+r_2r_4s_2+r_4s_3}}$$

$$\times\frac{(\lambda)_s\,v^s\,(-1)^{(p+q+u+v+1)r}}{s!\;(\mu x_1^{r_4})^r}\frac{(\lambda_1)_{s_1}\,\mu_1^{s_1}\,x_2^{r_1s_1}\,(-1)^{(p+q+u+v+1)r_1}}{s_1!\;(\mu x_1^{r_4})^{r_1s_1}}$$

$$\times\frac{(\mu_2x_2^{r_2})^{s_2}\,(-1)^{\{(p+q+u+v+1)r_2+p+q\}s_2}}{s_2!\;(\mu x_1^{r_4})^{s_2}}\frac{\mu_3^{s_3}\,(-1)^{\{(p+q+u+v+1)+p+q\}s_3}}{s_3!\;(\mu x_1^{r_4}x_3)^{s_3}}$$

$$=M_1\frac{(nr_4)!}{(2nr_4)!}\delta^{nr_4}x_1^{2nr_4}F^{2+q+v:h:m:1:1}_{1+p+u:k:w}\left[\begin{matrix}[-n:r,r_1,r_2,1]\\ [-nr_4:r,r_1,r_2,1]\end{matrix}\right.$$

$$[2nr_4:r,r_1,r_2,1];[(1-(B_q)-n):r,r_1,r_2-1],$$

$$\overline{\qquad\qquad\qquad};[(1-(A_p)-n):r,r_1,r_2-1],$$

$$[(1-(D_v)-n):r,r_1,r_2,1],[(E_h):1],[(G_m):1],[\lambda:1],[\lambda_1:1],$$

$$[(1-(C_u)-n):r,r_1,r_2,1],[(F_k):1],[(H_w):1],\text{—},\text{—},$$

$$\frac{(-1)^{r(p+q+u+v+1)}v}{(\mu x_1^{r_4})^r},\frac{(-1)^{r_1(p+q+u+v+1)}\mu x_2^{r_1}}{(\mu x_1^{r_4})^{r_1}},$$

$$\left.\frac{\mu_2x_2(-1)^{(p+q+u+v+1)r_2+p+q}}{(\mu x_1^{r_4})^{r_2}},\frac{\mu_3(-1)^{(p+q+u+v+1)+p+q}}{(\mu x_1^{r_4}x_3)}\right] \quad \ldots (3.7)$$

**Particular Cases**

(i) On making the substitution $p = 0 = q = u = v = r = s$; $\lambda_1 = 1 + \lambda_2$; $\lambda = 1 = v = r = r_4 = \mu = \mu_1 = x_2$, $x_1 = y_1$, in (3.7), we get

$$A_n^{\lambda_2}(y)=\frac{n!}{(2n)!}\frac{d^n}{dx^n}\left\{\frac{y^n}{1}F\left[\begin{matrix}1+\lambda_2;2n; & \\ & \frac{1}{y}\\ \text{—}; & \end{matrix}\right]\right\}$$

where $A_n^{\lambda_2}(y)$ are the Srivastava Polynomials[11].

(ii) On taking $p = 0 = q = v = s$; $u = 1 = r = s_1 = \lambda = v = \lambda_1 = r_4 = \mu = \mu_1$; $c_1 = 1 + \lambda$, and $\frac{1}{y}$ for $x$, in (3.7), we achieve

$$A_n^{\lambda}(y)=\frac{n!}{(2n)!}\frac{(1+\lambda)_n(-1)^n}{n!}\frac{d^n}{dx^n}\left\{y^{-n}F\left[\begin{matrix}—;-2n; \\ \\ -\lambda-n;\end{matrix}\ -y\right]\right\}$$

where $A_n^{(\lambda)}(y)$ are the Srivastava's Polynomials[12].

(iii) On setting $p = 0 = q = v = s$; $u = 1 = r = r_4 = s_1 = \lambda = \nu = \lambda_1 = \mu = \mu_1$; $c_1 = -\lambda$ and writing $\frac{1}{x}$ for $x_1$, in (3.7), we arrive at

$$f_n(y)=\frac{n!}{(2n)!}\frac{(-\lambda)_n}{n!}\frac{d^n}{dx^n}\left\{x^{-n}F\left[\begin{matrix}—;-2n; \\ \\ 1+\lambda-n;\end{matrix}\ x\right]\right\}$$

where $f_n(x)$ are the Pseudo-Laguerre Polynomials[13].

(iv) On setting $p = 0 = q = u = v = s$; $\lambda = 1 = \nu = \lambda_1 = r = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$ and writting $x$ for $x_1$ in (3.7) we achieve

$$H_n(x)=\frac{n!2^n}{(2n)!}\frac{d^n}{dx^n}\left\{x^nF\left[\begin{matrix}-\frac{n}{2},-\frac{n}{2}+\frac{1}{2};2n; \\ \\ —;\end{matrix}\ -\frac{1}{x^2}\right]\right\}$$

where $H_n(x)$ are the Hermite Polynomials.

**D. For $r_2 = 1$ and $r_3 = 1$**

Since $\delta^{nr_4}\left[\left(x_1^{r_4}\right)^{2n-r-r_1s_1-s_2-s_3}\right]$

$$=\frac{(2nr_4-rr_4-r_1r_4s_1-r_4s_2-r_4s_3)!\left(x_1^{r_4}\right)^{n-r-r_1s_1-s_2-s_3}}{(nr_4-r_4r_1-r_1r_4s_1-r_4s_2-r_4s_3)!}$$

where, $\delta\equiv\frac{\delta}{\delta x_1}$

Hence from (1.2), we have

$$R_n(x_1,x_2,x_3)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{1}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{1}\right]}\frac{[(A_p)]_{n-r-r_1s_1}}{[(B_q)]_{n-r-r_1s_1}}$$

$$\times\frac{[(C_u)]_{n-r-r_1s_1-s_2-s_3}\,[(E_h)]_{s_2}\,[(G_m)]_{s_3}\,(\lambda)_s\,\nu^s\,(\lambda_1)_{s_1}\,\mu_1^{s_1}}{[(D_v)]_{n-r-r_1s_1-s_2-s_3}\,[(F_k)]_{s_2}\,[(H_w)]_{s_3}\quad s!\quad s_1!\quad s_2!}$$

$$\times\frac{\mu_2^{s_2}\quad x_2^{r_1s_1+s_2}\quad \mu_3^{s_3}\quad \mu^{n-r-r_1s_1-s_2-s_3}}{s_3!\quad x_3^{s_3}\quad (n-r-r_1s_1-s_2-s_3)!}$$

$$\times\frac{(nr_4-rr_4-r_1s_1r_4-r_4s_2-r_4s_3)!\ \delta^{nr_4}x_1^{(2n-r-r_1s_1-s_2-s_3)}}{(2nr_4-r_4r_1-r_1r_4s_1-r_4s_2-r_4s_3)!} \qquad \ldots (3.8)$$

Now, since

$$\delta^{nr_4}\left(x_1^{r_4}\right)^{2n-r-r_1s_1-s_2-s_3}=0$$

for $\frac{n}{r}, \frac{n}{r_1}$, $n < k \leq 2n$ , hence after little simplification (3.8) can be thrown into the from

$$R_n(x_1, x_2, x_3) = M_1 \frac{(nr_4)!}{(2nr_4)!} \delta^{nr_4} x_1^{2nr_4} F^{2+q+v:h:m:1:1}_{1+p+u:k:w}$$

$$\left[\begin{array}{l}[-n:r,r_1,1,1],[2nr_4;r,r_1,1,1];\left[(1-(B_q)-n):r,r_1\right],\left[(1-(D_v)-n):r,r_1,1,1\right],\\ [-nr_4:r,r_1,1,1]\text{———};\left[(1-(A_p)-n):r,r_1\right],\left[(1-(C_u)-n):r,r_1,1,1\right],\end{array}\right.$$

$$[(E_h):1],[(G_m):1],[\lambda:1],[\lambda_1:1],$$

$$[(F_k):1],[(H_w):1],—,—,$$

$$\frac{(-1)^{r(p+q+u+v+1)r}}{(\mu x_1^{r_4})^r}, \frac{(-1)^{r_1(p+q+u+v+1)}\mu x_2^{r_1}}{(\mu x_1^{r_4})^{r_1}},$$

$$\left.\frac{\mu_2 x_2(-1)^{(p+q+u+v+1)+p+q}}{(\mu x_1^{r_4})}, \frac{\mu_3(-1)^{(2p+2q+u+v+1)}}{(\mu x_1^{r_4} x_3)}\right] \qquad \ldots (3.9)$$

**Particular Cases**

(i) On putting $p = 0 = q = v = s$; $r = u = 1 = x_1 = \lambda = v = r_4$; $\lambda_1 = -x$, , $C_1 = \beta_1 + x$ in (3.9) we achieve

$$m_n(x, \beta_1, c) = \frac{(\beta_1 + x)_n}{(2n)!} \frac{d^n}{dx^n} \left\{ F \left[ \begin{array}{ll} -n, -x; & \\ & \frac{1}{c} \\ 1 - \beta_1 - x - n; & \end{array} \right] \right\}$$

where $m_n(x; \beta_1, c)$ are the Meixner Polynomials.

(ii) On putting $p = 0 = q = u = v = h = s$; $k = 1 = \lambda = v = \mu_2 = \mu = r_4 = r_2 = x_1$; $x_2 = y$, $F_1 = 1 + \alpha$ in (3.9), we obtained

$$L_n^{\alpha}(y) = \frac{(1+\alpha)_n}{(2n)!} \frac{d^n}{dx^n} \left\{ F \left[ \begin{array}{ll} -n; & \\ & -y \\ 1+\alpha; & \end{array} \right] \right\}$$

where $L_n^{(\alpha)}(y)$ are the Laguerre Polynomials.

(iii) On making the substitution $p = 0 = q = u = h = s = r$; $r_2 = v = 1 = k = \lambda = v = \mu_2 = r_4$; $\mu = 1 = \mu_1$; $D_1 = 1 + \alpha$; $F_1 = 1 + \beta$; and instead of $x_2 = \frac{x-1}{x+1}$ in (3.9), we get

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\beta)_n}{(2n)!\; 2^n} \frac{d^n}{dx^n} \left\{ (x-1)^n F \left[ \begin{array}{ll} -n, -\alpha-n; & \\ & \frac{x+1}{x-1} \\ 1+\beta; & \end{array} \right] \right\}$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

(iv) If we take $p = 0 = q = u = h = s$; $v = 1 = k = \lambda = v = x_2 = r_2 = r_4$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = 1 + \beta$; $F_1 = 1 + \alpha$ and writing $\frac{x+1}{x-1}$ for $x_1$ in (3.9), we obtained

$$P_n^{(\alpha,\beta)}(x)=\frac{(1+\alpha)_n}{(2n)!\ 2^n}\frac{d^n}{dx^2}\left\{(x+1)^n F\left[\begin{matrix}-n,-\beta-n; & \\ & \frac{x-1}{x+1} \\ 1+\alpha; & \end{matrix}\right]\right\}$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

(v) When we take $p = 0 = q = u = h = s$; $v = 1 = k = \lambda = \nu = x_2 = r_2 = r_4$; $D_1 = 1 + \alpha = F_1$ and writing for $x_1$ in (3.9), we achieve

$$P_n^{(\alpha,\beta)}(x)=\frac{(1+\alpha)_n}{(2n)!\ 2^n}\frac{d^n}{dx^n}\left\{(x+1)^n F\left[\begin{matrix}-n,-\alpha-n; & \\ & \frac{x-1}{x+1} \\ 1+\alpha; & \end{matrix}\right]\right\}$$

where $P_n^{(\alpha,\alpha)}(x)$ are the Ultra spherical Polynomials.

(vi) If we take $p = 0 = q = u = v$; $h = 1 = k = \lambda = \mu = \nu = x = x_2 = r_2 = r_4$; $E_1 = 1 - z$; $F_1 = 2$; $\mu_2 = -2$ in (3.9), we have

$$g_{n+1}(z)=\frac{n!}{(2n)!}\frac{d^n}{dx^2}\left\{F\left[\begin{matrix}-n,-z; & \\ & 2 \\ 2; & \end{matrix}\right]\right\}$$

where $g_{n+1}(z)$ are the Mittage-Leffler Polynomials.

## Conclusion and Future Scope

In this article we have obtained many interesting new results for the generalized hypergeometric polynomial set $R_n(x_1,x_2,x_3)$ followed by important and interesting particular cases. Out of these particular results some of them stand for well known and some of them are believed to be new. These are of at most important for mathematicians' scientists, engineers and physical sciences, because the Rodrigues formulae are one of the most important tool for the generalization of various functions and polynomials.

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